PERFORMANCE EVALUATION OF RAINFALL-RUNOFF MODELS USING MULTI-OBJECTIVE OPTIMIZATION APPROACH

YOICHI FUJIHARA

Japan Science and Technology Agency
Gokasho, Uji, Kyoto 611-0011, Japan

HARUYA TANAKAMARU

Graduate School of Science and Technology, Kobe University
1-1 Rokkodai, Nada-ku, Kobe 657-8501, Japan

TAKESHI HATA, AKIO TADA

Faculty of Agriculture, Kobe University
1-1 Rokkodai, Nada-ku, Kobe 657-8501, Japan

We explore the effectiveness of multi-objective optimization approach for performance evaluation of rainfall-runoff models. The multi-objective optimization of the Tank Model is investigated using historical data of the Eigenji Dam Basin. *RMSE* (root mean square error) that emphasizes the error at high flows and *RR* (root mean square of relative error) that emphasizes the error at low flows are used as objective functions and these functions are simultaneously minimized. The multi-objective ES that combines the Evolution Strategy (ES) with the Pareto ranking and the MOCOM-UA are applied to this problem. Results show that the ES is superior to the MOCOM-UA in the accuracy of Pareto-optimal solutions. The multi-objective Tank Model optimization using the ES is also applied to the Osako and Syorenji Dam Basin. Moreover, the multi-objective optimizations of 2-layer, 3-layer and 4-layer Tank Model are carried out. Results indicate that the parameter set suited for an analysis purpose can be rationally selected by using shape of Pareto-optimal solutions in objective space. Results also show that the inadequacy and limitation of rainfall-runoff models can be identified and the performance evaluation of rainfall-runoff models can be done by using information obtained from multi-objective optimization approach.

INTRODUCTION

Conceptual rainfall-runoff models such as the Tank Model contain many model parameters, whose values are usually determined so that the model response closely matches the observed data. In this determination, single-objective optimization methods, which minimize a given objective function that measures the difference between model outputs and observed data, are widely used. However, the optimized parameters and model outputs considerably change according to selected objective function. Therefore, in the case of using the objective function that emphasizes the goodness of fit at high

flows, the model is able to simulate high flows with good accuracy but may be unable to simulate low flows well and vice versa. Another problem is that the single-objective optimization approach is inadequate to calibrate multi-output hydrologic models.

Recently, multi-objective optimization methods that optimize several objective functions simultaneously have been applied to the hydrologic model calibration. In the multi-objective optimization, it is needed to obtain the entire set of Pareto-optimal solutions. Although many optimization methods have been proposed to obtain Pareto-optimal solutions, the accuracy of solutions has seldom been examined and practical advantages of multi-objective approach have rarely been discussed.

In this study, the multi-objective optimization of the Tank Model is investigated using historical data of three dam basins. The multi-objective ES and the MOCOM-UA are applied, and the accuracy of Pareto-optimal solutions is examined. Moreover, the multi-objective optimizations of 2-layer, 3-layer and 4-layer Tank Model are also carried out, and the effectiveness of multi-objective optimization approach for performance evaluation of rainfall-runoff models is discussed.

HYDROLOGICAL DATA AND RAINFALL-RUNOFF MODEL

The Eigenji Dam Basin (131.5 km^2), Osako Dam Basin (114.8 km^2) and Syorenji Dam Basin (100.0 km^2) in Japan were chosen for study basins. The hydrological data consist of daily mean inflow (runoff depth) at the dam site, daily areal precipitation and evapotranspiration. The data of 4-water-year was used for model calibration.

The Tank Model (Figure 1) is a conceptual rainfall-runoff model developed by Sugawara. The model for long-term simulation has 16 parameters; side outlets a_1-a_5, bottom outlets b_1-b_3, side outlet heights from the bottom of each of the upper three tanks Z_1-Z_4 and initial storage depths S_1-S_4, which should be determined by the user. The feasible parameter space is defined by the upper and lower bounds of parameters.

MULITI-OBJECTIVE OPTIMIZATION OF THE TANK MODEL

Multi-objective optimization problem and classical method

The multi-objective optimization problem with the decision variable (parameter) vector $\boldsymbol{x}$ = $(x_1, x_2, \cdots, x_n)$ and the objective function vector $\boldsymbol{f}(\boldsymbol{x})$ can be described as follows.

$$\underset{\boldsymbol{x} \in X}{\text{minimize}} \; \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x})) \tag{1}$$

where m is the number of objective functions and X is the feasible region. The solution of this problem is referred to as Pareto-optimal solutions, whose mathematical definition is as follows. A decision variable vector $\boldsymbol{x}^1 \in X$ is said to dominate other decision variable vector $\boldsymbol{x}^2 \in X$, if $\forall j = 1, \cdots, m : f_j(\boldsymbol{x}^1) \leq f_j(\boldsymbol{x}^2)$ and $\exists j = 1, \cdots, m : f_j(\boldsymbol{x}^1) < f_j(\boldsymbol{x}^2)$. The decision variable vectors that are not dominated within the entire feasible region are denoted as Pareto-optimal solutions.
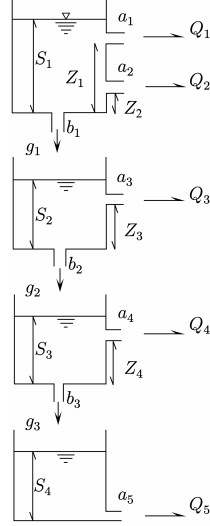
Figure 1. The Tank Model.

One of the classical methods for solving the multi-objective problem is the weighting method. In this method, each objective function is allocated a weight, and the multi-objective problem is converted into a single-objective problem as follows.

$$\underset{\boldsymbol{x} \in X}{\text{minimize}} \sum_{j=1}^{m} w_j f_j(\boldsymbol{x}) \tag{2}$$

where $w_j \geq 0$ and $w_1 + w_2 + \cdots + w_m = 1$. One Pareto-optimal solution can be obtained by giving a weight parameter set. However, in this method, the huge amount of calculation is required to obtain the entire set of Pareto-optimal solutions.

Global multi-objective optimization methods

The multi-objective ES [Fujihara *et al*. [1]] and the MOCOM-UA [Yapo *et al*. [2]] are investigated. The multi-objective ES is the method that combines the Evolution Strategy (ES) [Schwefel [3, 4]] with the Pareto ranking. The main feature of the ES is that its principal search procedure is mutation, which is formulated as follows.

$$x_i'' = x_i' + \sigma_i'' N_i(0,1) \tag{2}$$

$$\sigma_i'' = \sigma_i' \exp(\tau' N(0,1) + \tau N_i(0,1)) \tag{3}$$

where $i = 1, \cdots, n$, n is the number of decision variables, σ_i are mutation parameters, $N_i(0,1)$ are normal random variables for each decision variable, $N(0,1)$ is a normal random variable and τ' and τ are operation parameters; $\tau' \propto (2n)^{-1/2}$ and $\tau \propto (2n^{1/2})^{-1/2}$. The self-adaptation strategy is introduced to automatically update the standard deviations σ_i, which are treated as gene as well as the decision variables x_i. Therefore, each individual $\boldsymbol{a}$ is represented as $\boldsymbol{a} = (\boldsymbol{x}, \boldsymbol{\sigma})$. The recombination is formulated as follows.

$$x_i' = x_{k_1,i} + \chi_i(x_{k_2,i} - x_{k_1,i}) \tag{5}$$

$$\sigma_i' = \frac{1}{\mu}\sum_{k=1}^{\mu} \sigma_{k,i} \tag{6}$$

where $x_{k_1,i}$ and $x_{k_2,i}$ are two parent individuals selected at random from the population, χ_i is a uniform random variable over [0, 1] and μ is the number of the population.

The Pareto ranking [Goldberg [5]] is adopted as a fitness evaluation. In this ranking, all the nondominated individuals in the population are assigned the rank 1. These individuals are set aside, and the nondominated individuals in the remaining population are assigned the rank 2. In the selection procedure of the ES, individuals which have the small order of this rank are preferentially selected.

The MOCOM-UA is a general-purpose global multi-objective optimization algorithm, which combines the controlled random search with the competitive evolution, Pareto ranking, and newly developed strategy of multi-objective downhill simplex search. Iterative application of the ranking and evolution procedures makes the entire population to converge toward the Pareto-optimal solutions. The procedure terminates automatically when all individuals in the population become nondominated.

Application conditions

Multi-objective optimization of the Tank Model is investigated using historical data of the Eigenji Dam Basin. *RMSE* (root mean square error) that emphasizes the error at high flows and *RR* (root mean square of relative error) that emphasizes the error at low flows were used as objective functions and these functions are simultaneously minimized.

$$RMSE = \frac{1}{Q_m}\sqrt{\frac{1}{N}\sum_{t=1}^{N}(Q_{ct}-Q_{ot})^2} \tag{7}$$

$$RR = \sqrt{\frac{1}{N}\sum_{t=1}^{N}\frac{(Q_{ct}-Q_{ot})^2}{Q_{ot}^2}} \tag{8}$$

where N is the number of data, Q_m is the mean observed daily runoff depth, and Q_{ct} and Q_{ot} are the calculated and observed daily runoff depths, respectively. The water balance constraint given in Eq. (9) was introduced by using a penalty function.

$$\left|\sum_{t=1}^{N}Q_{ct} - \sum_{t=1}^{N}Q_{ot}\right| \Big/ \sum_{t=1}^{N}Q_{ot} < \varepsilon \tag{9}$$

where ε is the permitted error of the water balance. ε was set equal to 0.02 here.

The strict Pareto-optimal solutions of this problem are obtained by the weighting method. The converted single-objective problem is solved by using the SCE-UA [Duan *et al.* [6]]. The solutions are compared with those of the ES and the MOCOM-UA, and the accuracy of these solutions is examined. In the application of the weighting method, 100

optimization runs with different initial populations were conducted for one of the weight parameters and the run of minimum function value was adopted. The weight parameter was set from 0.0 to 1.0 at 0.05 intervals. Consequently, it was needed about 21×10^7 function evaluations to obtain 21 strict Pareto-optimal solutions.

Results

In the application of the multi-objective ES, when the number of function evaluations reached 10^6, the multi-objective optimization run was terminated and 237 Pareto-optimal solutions were obtained. In the application of the MOCOM-UA, a population size of 500 was selected [Yapo *et al*. [2]]. The MOCOM-UA required about 8×10^5 function evaluations until all individuals are assigned the rank 1.

Figure 2 illustrate the relationship between *RMSE* and *RR* in objective space obtained by three multi-objective optimization methods. Results show that Pareto-optimal solutions by the multi-objective ES are almost the same as those by the weighting method and a large number of Pareto-optimal solutions are obtained by relatively small amount of calculation. Moreover, some parts of Pareto-optimal solutions are not obtained by the weighting method but the multi-objective ES solutions are obtained continually. On the other hand, The MOCOM-UA estimates 500 Pareto-optimal solutions. However, almost all solutions converge into one part and the entire Pareto-optimal solutions cannot be obtained.

In terms of the accuracy of Pareto-optimal solutions and computational efficiency, the multi-objective ES is superior to the MOCOM-UA and is considered to be effective and efficient in multi-objective optimization of rainfall-runoff model parameters.
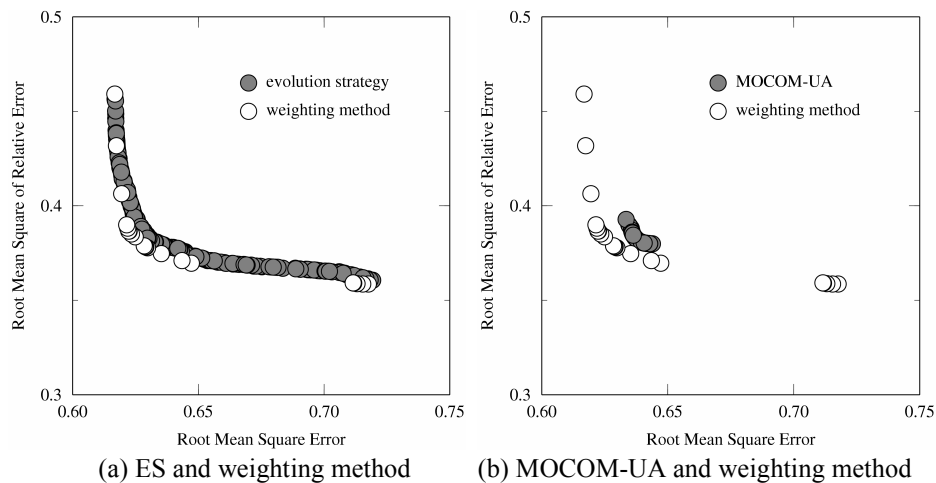

(a) ES and weighting method   (b) MOCOM-UA and weighting method

Figure 2. Relationship between *RMSE* and *RR* of Pareto-optimal solutions.

PERFORMANCE EVALUATION OF RAINFALL-RUNOFF MODELS

Application conditions

In addition to the Eigenji Dam Basin, the multi-objective Tank Model optimization is applied to the Osako and Syorenji Dam Basin. Furthermore, the multi-objective optimizations of 2-layer (8 parameters) and 3-layer Tank Model (12 parameters) are carried out using historical data of the Eigenji Dam Basin and, results are compared with the 4-layer case (16 parameters). The objective functions and constraints are the same as those of the previous section. Here, parameters are optimized by using the multi-objective ES, which showed the best performance in solving multi-objective problems.

Results

The relationship between *RMSE* and *RR* for the Eigenji, Osako and Syorenji Dam Basin is shown in Figure 3. *RR* and *RMSE* have typical trade-off relationship, and the parameter set that simultaneously minimizes both objective functions cannot be obtained. In the case of the Osako Dam Basin, in order to make the value of *RR* smaller than about 0.23, the value of *RMSE* has to be very large. On the other hand, in the case of the Syorenji Dam Basin, in order to make the value of *RMSE* smaller than about 0.84, the value of *RR* has to be very large. In such cases, if we desire the model parameter set that gives good agreement of observed and simulated hydrographs across all flow ranges, the parameter set corresponding to the bending point of the Pareto-optimal solutions in objective space should be selected in order to avoid the extreme degradation of *RR* or *RMSE*. Thus, the parameter set suited for an analysis purpose can be selected rationally by using shape of Pareto-optimal solutions in the objective space.

The relationship between *RMSE* and *RR* for 2-layer, 3-layer and 4-layer Tank Model is shown in Figure 4. In the case of the 3-layer and 4-layer Tank Model, if we desire the model parameter set that gives good fit at all flow ranges of hydrograph, the parameter
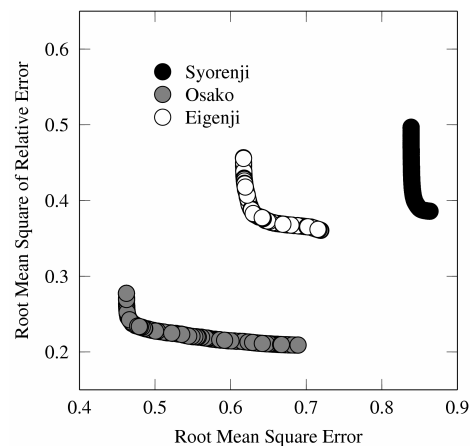

Figure 3. Relationship between *RMSE* and *RR*: Eigenji, Osako and Syorenji.

set corresponding to the bending point of the Pareto-optimal solutions can be selected as same as the above example (Figure 3). However, in the 2-layer Model, it is difficult to select such a solution because the shape of the Pareto-optimal solutions in objective space is almost straight and there is not a bending point. Therefore, the model that can simulate both low and high flows with good accuracy cannot be obtained in the 2-layer case.

The hydrograph ranges associated with the Pareto-optimal solutions for 2-layer and 4-layer Tank Model are presented in Figure 5 and Figure 6, respectively. The hydrograph range of 2-layer is extremely larger than that of 4-layer. Especially, the hydrograph range of 2-layer is extremely large at low flows. The observed hydrograph is not within the range and the simulated hydrograph underestimates continuously. The range of 3-layer not shown here is almost similar to that of 2-layer. From these results, it is found that 2-layer and 3-layer Tank Model are not suited to simulate hydrographs in this basin. Thus, the inadequacy and limitation of rainfall-runoff models can be identified and the performance evaluation of rainfall-runoff models can be conducted by using multi-objective approach.

CONCLUSIONS

In this study, the multi-objective optimization of the Tank Model was carried out and the effectiveness of multi-objective optimization approach for performance evaluation of rainfall-runoff models was investigated. The results are summarized as follows: (1) The multi-objective ES is superior to the MOCOM-UA in terms of the accuracy of Pareto-optimal solutions and is effective and efficient in multi-objective optimization of rainfall-runoff model parameters. (2) The shape of Pareto-optimal solutions in objective space makes it possible to select the rational parameter set suited for an analysis purpose. (3) Using multi-objective optimization approach, the inadequacy and limitation of rainfall-runoff models can be identified and the performance evaluation of rainfall-runoff models can be performed effectively.
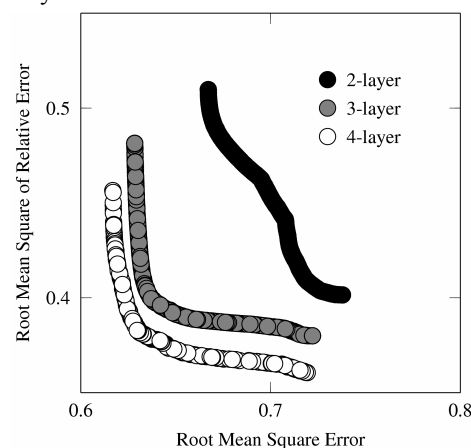

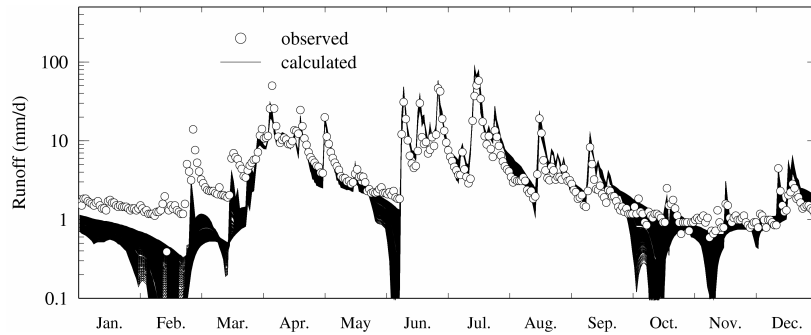Figure 4. Relationship between *RMSE* and *RR*: 2-layer, 3-layer and 4-layer.

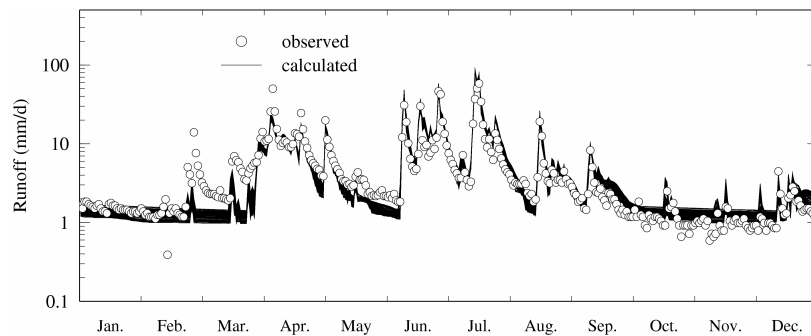Figure 5. Simulated daily runoff hydrograph: 2-layer, Eigenji (1984).

Figure 6. Simulated daily runoff hydrograph: 4-layer, Eigenji (1984).

REFERENCES

[1] Fujihara Y., Tanakamaru H., Tada A. and Hata T., "Calibration of Rainfall-Runoff Models Using the Evolution Strategy", *Proceedings of the 1st International Conference on Hydrology and Water Resources in Asia Pacific Region*, Vol. 2, (2003), pp 885-890.

[2] Yapo P. O., Gupta H. V. and Sorooshian S., "Multi-objective global optimization for hydrologic models", *Journal of Hydrology*, No. 204, (1998), pp 83-97.

[3] Schwefel H.-P., "*Numerical Optimization of Computer Models*", Birkhauser, (1977).

[4] Schwefel H.-P., "*Evolution and Optimum Seeking*", John Wiley & Sons, (1995).

[5] Goldberg D. E., "*Genetic Algorithms in Search, Optimization, and Machine Learning*", Addison-Wesley, (1989).

[6] Duan Q., Sorooshian S. and Gupta V. K., "Effective and efficient global optimization for conceptual rainfall-runoff models", *Water Resources Research*, Vol. 28, No. 4, (1992), pp 1015-1031.